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THESIS

A SPARES OPTIMIZATION MODEL FOR DEPLOYABLE
U.S. MARINE CORPS UNITS

by

Paul R. Yorio

March 1988

Thesis Advisor: Donald P. Gaver

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A model is developed which provides the operational commander with a stockage policy for spare secondary reparables (e.g., tank engines, amtrack transmissions, etc.) that optimizes the probability of successful mission completion subject to weight or volume constraints imposed by the MAGTF's mode of deployment. Optimization of this stockage policy is stochastically modeled using data from the Marine Corps Integrated Maintenance Management System data base and then solved as an integer program. (Continued)

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A Spares Optimization Model for Deployable U.S. Marine Corps Units

by

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Submitted in partial fulfillment of the requirements for the degree of

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ABSTRACT

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The integer program is coded using the Generalized Algebraic Modeling System (GAMS) language and solved using the Zero/One Optimization Methods mixed integer program solver. Operational data for a Marine Amphibious Unit yields an integer program with 190 binary variables and 26 constraints. A solution within 0.07% of optimality is obtained on an IBM 3033AP computer in 3.9 seconds and on a Zenith Z-248 personal computer in 176 seconds.

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I. INTRODUCTION

The combat structure of the United States Marine Corps (USMC) centers around the concept of the Marine Air-Ground Task Force (MAGTF) which is a combined arms force consisting of ground, air, and combat service support forces. The size of a MAGTF and its specific composition are dependent upon the particular mission assigned and the capability of the opposing forces. However, all MAGTFs are organized to take maximum advantage of the combat potential inherent in a rapidly deployable, closely integrated air-ground team under the control of a single commander. A MAGTF consists of four basis elements: command element, ground combat element, aviation combat element and combat service support element. It is the mission of the combat service support element to provide all logistical support for the ground and aviation combat elements. [Ref. 1:pp.1-4].

In this study, a model is developed which provides the operational commander with a stockage policy for spare secondary reparables (e.g., tank engines, amtrack transmissions, etc.) that optimizes the probability of successful mission completion within the constraints imposed by the MAGTF's mode of deployment (i.e., airlift or sealift). The problem of determining this optimal stockage

policy is stochastically modeled using operational data resident in the Marine Corps Integrated Maintenance Management System (MIMMS) data base and then solved as a mathematical program. The remainder of this chapter will include a description of the scenarios which are of particular interest to the U.S. Marine Corps, an explanation of terms, and a definition of the problem. Additionally, justification for the approach taken here as compared to those in standard inventory models is provided.

A. PROBLEM STATEMENT

The U.S. Marine Corps currently deploys MAGTFs by airlift or sealift to participate in numerous short-term exercises such as BRIGHT STAR in Egypt, GALLANT EAGLE in the southwestern United States, and COLD WINTER in Norway. These MAGTFs range in size from 2,000 man Marine Amphibious Units (MAUs) to 16,000 man Marine Amphibious Brigades (MABs), depending upon the particular exercise. These exercises are of such duration that resupply of the MAGTF's ground combat element by strategic airlift or sealift from logistics bases in the continental United States is not practical. Furthermore, by doctrine [Ref. 1:pp. 1-4], a MAGTF is task organized to maximize combat power; therefore, the number of mechanics, technicians, and other logistics personnel together with their test and repair equipment are kept to a minimum. Thus, the quantities of spare repair

parts stocked by the combat service support elements are all that are available to support the requirements of the ground combat element for the duration of the exercise.

In addition to these operational exercises, the Marine Corps deploys Marine Amphibious Units (MAUs) to the Mediterranean Sea as part of the U.S. Sixth Fleet on a continuous, six month rotating basis. These Mediterranean MAUs remain aboard ship except when conducting exercises. Therefore, the inventory of spares maintained by their combat service support element, called a MAU Service Support Group (MSSG), is limited by the cargo capacity of the assigned amphibious ships. Even though resupply from the continental United States by air is possible, it is desired to reduce dependency on this costly method [Ref. 2:p. 1]. Thus, similar to the above exercise scenarios, the MSSG must stock sufficient spares, within the capacity constraints of the assigned shipping, so as to satisfy the anticipated demands generated by the ground combat element's participation in exercises during the course of the six month deployment.

Presently, a problem of even greater concern to Marine Corps combat service support organizations than either of those previously mentioned is the Marine Corps's role in the rapidly evolving maritime strategy of the U.S. Navy. As the amphibious power projection arm of the Navy, the Marine

Corps, in conjunction with the Navy, has developed and recently implemented the concept known as the Maritime Prepositioning Force (MPF). According to doctrine [Ref. 3], a MPF consists of a squadron of four to five ships loaded with combat equipment and supplies. One such MPF squadron is presently located in each of the following three ocean areas: Eastern Atlantic, Indian Ocean,and Western Pacific. When a MPF operation is ordered, a fly-in echelon consisting of 16,500 Marines and sailors, comprising a Marine Amphibious Brigade (MAB) and a Navy Support Element, is airlifted to a benign airfield in the vicinity of the objective area for link-up with equipment and supplies aboard the MPF squadron ships. The Brigade can be combat-capable and ready to move to an objective in five days or less and can operate for thirty days, independent of any strategic airlift or sealift from the United States, using the supplies and spare parts inventories on the ships [Ref. 4:p. 11]. It is this last requirement, to operate independently for thirty days using organic supplies and inventories, that poses the greatest challenge to Marine Corps combat service support planners. However, as with supporting short-term exercises and MAU deployments, the problem of supporting the MPF MAB simplifies to one of stocking adequate spare parts inventories, subject to the capacity constraints of the MPF squadron ships, to meet the

demand during the MAB's projected missions. The fact that such demand may vary unpredictably must be confronted.

At the present time, to solve spare stockage problems similar to those described above, Marine Corps combat service support planners determine the number of secondary items (referred to in Marine Corps' terminology as *secondary reparables*) to stock for each principal end item in the particular MAGTF's Table of Equipment (T/E) by using either last-period demand method or a simple moving average of past or historical demands generated during similar exercises or Mediterranean deployments. In the situation where no historical demands are available, such as with the new MPF MABs, stockage levels are determined using estimated replacement rates in combination with the experience of maintenance and supply personnel. These techniques have proved inadequate [Ref. 2:p. 1] because they fail to consider such factors as changes to a MAGTF's Table of Equipment and variations in the operational schedule, both of which may occur from one exercise or deployment to the next. More importantly, they neglect to consider that the demands placed on the supply system are the direct result of equipment failures.

To overcome these inadequacies, it is the objective of this study to develop a model which provides the operational commander at the MAU or MAB level with a stockage policy for

spare secondary reparables that optimizes the probability of successful mission completion within the constraints imposed by the mode of deployment. The proposed model is sufficiently general so as to be adaptable to any of the above scenarios and it can accommodate any additions or deletions to the MAGTF's Table of Equipment or variations in mission duration among any combination of principal end items. Additionally, the model incorporates a measure of effectiveness which provides the operational commander with an indicator of the overall impact of the stockage policy on successful mission completion. Furthermore, the model utilizes the operational maintenance data resident in the Marine Corps Integrated Maintenance Management System (MIMMS) to determine operationally based failure rates. Thus, by use of this model a nearly optimal stockage policy for secondary reparables at the retail level can be determined for a wide variety of inventory problems currently facing Marine Corps combat service support planners. This will result in a more effective utilization of the limited space and weight resources available for spares' inventories on shipping or aircraft assigned to deploying Marine Corps units.

B. INVENTORY MODELS AND SOLUTION APPROACHES

The stockage problems confronting Marine Corps planners can all be viewed as variations of the single period, constrained, multi-item inventory problem. The simplest such single-period model is the classic newsboy problem [Ref. 5], alternatively referred to as the Christmas tree problem. The characteristic distinguishing the models of this thesis from other standard inventory models, such as the classical economic-order-quantity or economic-order-interval models [Ref. 6:pp. 159-289] is that only a single time period, usually of finite length, is relevant; therefore, only a single initial procurement or order is made [Ref. 6:p. 297]. The previously described scenarios all fall within the context of single-period models because the demand for secondary reparables is the direct result of equipment failures which occur at infrequent intervals during the course of the exercise. Resupply either does not occur or is kept to a minimum, and thus it can be ignored. Furthermore, it is Marine Corps policy to permit only functional unit replacement at the field or organizational levels of maintenance. Actual repair of secondary reparables at the intermediate maintenance level is very limited and for all practical purposes performed only at the depot level. Therefore, since the MAGTFs of concern in these scenarios are usually only authorized up to the intermediate

level of maintenance, the actual repair and return of secondary reparables to these retail levels of inventory is insignificant and can be ignored. This is particularly true when one considers that some of the exercise durations are of the same order of magnitude as the average repair time for some of the secondary reparables. Thus, each scenario consists of an exercise (or exercises) which is (are) of finite duration and during which repair and resupply are essentially nonexistent, so the combat service support planner must make a one-time decision of what to stock, not how much of each item to procure. He must do so within the constraints of the mode of deployment. This is analogous to the single period, constrained, multi-item inventory problem.

There are several examples in the literature of single period inventory models applied to military logistics situations. They include the spare parts kit problem [Ref. 7:pp. 281-295], the fly-away kit problem [Ref. 6:p. 324], and the submarine provisioning problem [Refs. 8:pp. 236-243, 9:p. 407]. The problem with each of these military applications is that they assume that there is some monetary stockout cost incurred when a demand exists and no spares are available to meet the demand. This is realistic in the civilian sector where real monetary values can often be associated with each stockout. But, in most practical

military applications it is difficult, if not impossible, to determine a tangible, realistic, and objective stockout cost. This is particularly true for each of the previously described scenarios because there is no way to place a cost on the inability to complete a mission because of a deficiency of spares. Thus, it is difficult to apply the models existing in the literature to these real-world Marine Corps inventory problems.

This problem is not insurmountable. In the civilian sector when an organization does not know its stockout costs or is not confident in its estimates of them, management will set *service levels* which indicate the ability to meet customer demands from stock [Ref. 10:p. 149]. A service level is basically the complement of the probability of stockout and really represents a subjective decision by management to accept some degree of stockout or customer disservice [Ref. 10:p. 150]. The U.S. Navy actually uses a similar concept when establishing its Aviation Consolidated Allowance Lists (AVCAL) and Coordinated Shipboard Allowance Lists (COSAL) [Ref. 11:pp. 4-30-4-50]. The service level measure is less than satisfactory for military applications since it is subjectively established by combat service support planners. Service levels do not provide the operational commander in the field with a meaningful indicator of the impact of the recommended stockage policy

on successful mission completion. This is because there is no direct relationship between the probability of stockout and the probability of successfully completing a given mission. A high service level does not equate to an optimal probability of mission completion. This is particularly true in the constrained, multi-item inventory situation where it might actually be better to accept a greater probability of stockout (lower service level) for a secondary reparable with a low failure rate in order to be able to stock more of another secondary reparable with a high failure rate.

To overcome the disadvantages associated with the minimization of stockout costs (actually variable costs) and service level approaches, the proposed model introduces a new measure of effectiveness, the *probability of mission completion given the specified stockage policy,* which will be referred to as the *probability of mission completion* throughout the remainder of this paper. This probability extends the familiar concept of reliability to include a system and its inventory of spare parts [Ref. 8:p. 237]. A system is defined to be the quantity of a principal end item required to be operational throughout the duration of a given mission. This end item must be a "readiness reportable pacing item" as delineated in Marine Corps Bulletin 3000 [Ref. 12:pp. 1-12 Encl (2)]. A readiness

reportable pacing item is a principal end item whose operation is essential to the ability of a MAGTF to complete its assigned mission. Simply stated, the probability of mission completion is the probability that k-out-of-n ($k \leq n$) readiness reportable pacing items of the same type remain operational for a specified mission with a given stockage policy. Black and Proschan [Ref. 7:p. 283] proposed a similar measure of effectiveness in their early work using marginal analysis to determine optimal spare parts kits at minimum cost. They called it *probability of adequacy* or *assurance of adequacy*, which is defined as

> The probability that either the equipment does not fail due to a. . .malfunction or, if it does fail for this reason, a replacement is available to provide continued successful equipment operation. [Ref. 8:p. 237]

However, this definition applies only to series systems in which all equipment must remain operational for a successful mission. The proposed probability of mission completion is more general since it can be extended to "k-out-of-n systems". This is required in the present study because the operational commander, not the military logistician, determines the quantities of a principal end item which he feels are necessary to complete the assigned mission. For example, a MAU commander may feel that he can accomplish a particular mission with just three out of five tanks which permits fewer tank secondary reparables to be stocked than if he required all five tanks to complete the same mission.

In this study the inventory problems in each of the previously described scenarios are modeled as single period, constrained, multi-item inventory problems in which the demands are the result of independent failures of secondary reparables. In the mathematical formulation of these models, the probability of mission completion is maximized, as opposed to the standard approach of minimizing expected (variable) costs. This provides models which reflect the fact that in most military situations, and specifically in the three Marine Corps scenarios studied here, stockout or shortage costs cannot be determined. The inclusion and numbers of constraints are dependent on the mode of deployment. However, in all cases studied, they are linear and will be referred to as capacity constraints. Thus, each model will be viewed as a mathematical program in which the objective function to be maximized is a nonlinear, nondecreasing function of the decision variable representing the quantity of secondary reparables to stock. Inclusion of the linear capacity constraint or constraints result in an overall integer nonlinear programming problem. Since the objective function is itself a complicated function of the decision variable, the solution approach is to linearize the objective function and then reformulate the entire problem using binary variables. This reformulation converts the original integer nonlinear programming problem into a binary

linear programming problem which is then coded for computer processing using the Generalized Algebraic Modeling System (GAMS) [Refs. 13, 14, 15] and solved using the Zero/One Optimization Methods (ZOOM) [Ref. 16] mixed integer program solver.

Since one of the primary purposes of this study is to provide the combat service support planners of the Second Force Service Support Group at Camp Lejeune, North Carolina with models solvable on a personal computer, it is desired to avoid dependency on commercial optimization packages not locally available. Therefore, the initial solution methodology utilized Langrangian relaxation to establish an upper bound on the optimal solution of the integer linear problem. Then a heuristic based on marginal analysis is used to improve upon the best feasible solution obtained during the establishment of this upper bound, and this heuristically improved solution is accepted as the final solution provided that its associated objective value was reasonably close to the upper bound. This methodology is similar to that implemented by DeWolfe [Ref. 17:pp. 28-32] to solve a Selective Reenlistment Bonus problem for the U.S. Marine Corps. The advantage of this approach is that it can be programmed in a language such as FORTRAN 77 thus avoiding dependency on a commercial optimizer. However, as the number of constraints increase the programming difficulty

associated with the search procedure used in establishing the best upper bound increases, and for any more than two constraints actually becomes prohibitive. So, to keep the solution approach as flexible and general as possible, it was decided to solve all integer linear problems with the GAMS/ZOOM optimization package (commerically available in March 1988), thus allowing the solution of models with more than one constraint and avoiding restrictive programming considerations.

The above solution approach overcomes the disadvantages associated with more standard solutions of the single period inventory model. For instance, it could be argued that since these inventory problems are all examples of decision making under risk, they could be solved using decision theory and payoff matrices to minimize expected costs. However, as was pointed out earlier, realistic monetary stockout costs are extremely difficult to determine in most military applications. Additionally, in the multiple item inventory problem, the payoff matrices become very large and the interrelationship among the payoffs associated with different combinations of items becomes difficult to evaluate and enumerate, so it is computationally more efficient to use a marginal analysis approach [Ref. 10:p. 269]. Marginal analysis provides the basis for the techniques used in the spare parts kit problem [Ref.

7:pp.283-284] and in the submarine provisioning problem [Ref. 8:p. 237], but it has practical limitations when more than one constraint must be considered. Hadley and Whitin [Ref. 6:pp 328-330] obtained exact integer solutions to their single constraint, fly-away kit problem using dynamic programming and Dreyfus and Law [Ref. 18:pp. 107-118] offer dynamic programming algorithms to solve single constraint, cargo-loading problems. However, they all point out that for problems involving more than two constraints even dynamic programming has computational limitations, particularly when both the number of constraints and decision variables are large [Refs. 6:p. 331, 18:pp. 107-118]. The solution approach utilized in this study avoids these shortcomings.

C. THESIS OUTLINE

This thesis develops and presents a method for determining spare stockage levels of secondary reparables associated with the readiness reportable principal end items of a given Marine Air-Ground Task Force. In Chapter II, the maintenance data base, data analysis, and problems encountered during this analysis are discussed. The objective functions are defined, developed, and justified in Chapter III. Additionally, each scenario is modeled and formulated as an integer nonlinear programming problem for

which the objective function is linearized so that the problem can be reformulated as a integer linear problem to facilitate its solution. Specific solution methodology is presented in Chapter IV. The initial approach using Lagrangian relaxation with a heuristic is briefly described before the more general procedure using GAMS/ZOOM is developed and implemented. In Chapter V results using actual data for a typical Marine Amphibious Unit are presented and verification of the model is conducted using different sets of simulated failure data. The chapter concludes with a discussion of computational experience for both the mainframe and personal computer. Chapter VI contains conclusions and recommendations. Finally, listings of both FORTRAN 77 and GAMS source codes are provided in the appendices.

II. DATA ISSUES AND ANALYSIS

A primary impetus for this study was the desire to develop a reliability-based, spares stockage model for secondary reparables utilizing the maintenance data resident in the Marine Corps Integrated Maintenance Management System (MIMMS). Currently, spare stockage models within the Marine Corps are limited to those based on forecasts from past demands generated against the supply system. This forecasting approach neglects to consider that the source of the demands is the failure of equipment such as secondary reparables, and it is precisely this type of failure information that can be extracted from relevant MIMMS sub-files which permits the development of a reliability-based model.

A. DESCRIPTION OF DATA

MIMMS is an automated system which accumulates maintenance data on all serialized, principal end items in the Marine Corps inventory. It records and accumulates, in various sub-files, maintenance actions performed on a serialized end item during its service life at each of the three levels of maintenance (i.e., organizational, intermediate, and depot) [Ref. 19]. In particular, the MIMMS sub-file of relevance to this study is the Equipment

Repair Order History File. This sub-file maintains all equipment repair orders (EROs) that were completed on every serialized end item during the past eighteen months, and it is updated on a quarterly basis. An ERO is opened each time a serialized end item is inducted into the maintenance cycle and contains information such as status of the end item (i.e., combat deadlined, safety deadlined, mission degraded, etc.), the meter reading at the time of the current maintenance action (expressed in either hours, miles, rounds or days depending on the end item), and the defect code which describes the nature of the failure. By looking at only those EROs in the MIMMS ERO History File which were opened for critical, combat deadlining repairs (i.e., category code M), a new, local user file can be created. This user file is organized by principal end item and includes only those end items which are considered "readiness reportable" [Ref. 12:pp. Encl (1) 1-23]. Associated with each end item are all serial numbers for which a combat deadlining ERO exists; each serial number has a meter reading and defect code to reflect each ERO completed on that particular serial number during the eighteen month period covered by the ERO History File. Obviously, there may be multiple entries under the same serial number indicating that this end item suffered more than one combat deadlining failure during the reporting

period. There may be no entries if no failures were experienced by the particular serial number. Table 1 provides a sample extract from this user-created data file and displays its contents for several tank serial numbers.

TABLE 1
EXTRACT FROM LOCAL USER DATA FILE

SERIAL #	EOTC*	METER READING	DEFECT CODE**
502814	H	74	D05 (POWER TRAIN,CLUTCH)
502828	H	51	A34 (ENGINE REPLACE)
	H	75	J34 (COOLING SYS REPLACE)
	H	107	C34 (POWER RACK REPLACE)
	H	155	A34 (ENGINE REPLACE)
	H	163	B34 (TRANSMISSION REPLACE)
502831	H	81	A34 (ENGINE REPLACE)
	H	149	B34 (TRANSMISSION REPLACE)
502835	H	62	K01 (ELEC SYS, GENERATOR)

* EOTC= Equipment Operating Time Code (H=Hours, M=Miles, R=Rounds, D=Days)

** Defect Codes are alphanumeric codes and have been annotated here for display purposes only.

The data in this consolidated file can be used to estimate failure rates for each secondary reparable in either of two ways and both methods will be described in the following section along with their advantages and disadvantages. It must be stated that both the format and method of update for many of the MIMMS files require improvement before a truly comprehensive analysis of the data is possible. A case in point is the ERO History File used in this study, and recommendations for its improvement are provided in the

following paragraph and in Chapter VI which minimize the impact on current MIMMS file management and the amount of effort required at the using unit input level.

One of the basic problems with the ERO History File is that potentially useful data is being discarded during each quarterly update in order to minimize file size. What is proposed here is to maintain a local (i.e., at the Force Service Support Group level) consolidated version of the ERO History File with a format similar to that described in the preceding paragraph. This file should be updated when its parent ERO History File is updated at the end of each quarter. Since this consolidated file contains only information extracted from combat deadlining EROs, which comprise a small fraction of the total EROs in the parent file, there is a greater capability to accumulate relevant data without the need to automatically discard the oldest three months of data. The basic algorithm for each quarterly update is:

Step 1. Select a combat deadlining ERO (Category Code M) from the ERO History File if one exists. If none exist, STOP.

Step 2. Determine if the serial number associated with this ERO is already resident on the consolidated file. If it is, continue with Step 3. If it is not, add the serial number and its associated meter reading and defect code to the consolidated file under the appropriate principal end item. Return to Step 1.

Step 3. Determine if the same defect code exists for this serial number. If it does, continue with Step 4. If it does not, add the meter reading and defect code to the list associated with this serial number. Return to Step 1.

Step 4. Subtract the two meter readings to obtain a time between failure for this type of defect. Add this new data point to the list associated with this type of defect. Delete the record associated with the older meter reading (i.e., the smallest meter reading) and add the new meter reading and defect code to the end of the list associated with this serial number. Return to Step 1.

The result is an updated version of the consolidated file which is retained at the local level for update during the next quarterly cycle. Each readiness reportable principal end item in the file is broken down by serial number which in turn contain records of the most recent meter readings and defect codes. Each end item also consists of lists containing times between failures for each type of defect. Thus this local consolidated file does not alter any existing MIMMS file definitions, nor does it necessitate the creation of any new system file definitions. More importantly, it provides the combat service support planner with a consolidation of all the pertinent information required to compute failure rates and track the life history of any secondary reparable associated with a readiness reportable end item. This is a significant improvement over the present method which is restricted to using the most recent eighteen months of failure data resident in the ERO

History File, and it is this restriction that limits the accumulation of failure times for certain types of secondary reparables (e.g., certain radio transmitters).

B. DATA ANALYSIS

Using the data resident in the consolidated file, the operating times between failures for each type of secondary reparable were computed. Failure rates were estimated using two different approaches. In the first approach, an exponential lifetime distribution was assumed for each secondary reparable and the maximum likelihood estimator (M.L.E.) for the parameter λ_i of an exponential distribution was used to provide an estimate of the failure rate. The second approach involves a Bayesian analysis of the times between failure to determine a posterior distribution for the failure rate of a secondary reparable.

In deriving the M.L.E. for λ_i, it is assumed that the observed lifetimes of each secondary reparable i form a relevant random sample of size n_i from an exponential distribution with unknown parameter λ_i. The likelihood function must also account for the exposure times during which no failures of type i occurred. If these terms are not considered, the computed failure rate will tend to be too large and excessive stockage levels would result. For the n_i observed failure times and the observed exposure time

t'_i during which no failures occurred, the likelihood function is:

$$f\left(t_i \mid \lambda_i\right) = \lambda_i \exp^{-\lambda_i t_{i1}} \bullet \ldots \bullet \lambda_i \exp^{-\lambda_i t_{in_i}} \bullet \exp^{-\lambda_i t'_i} = \lambda_i^{n_i} \exp^{-\lambda_i\left(\sum_{j=1}^{n_i} t_{ij} + t'_i\right)} .$$

Maximizing the likelihood function with respect to λ_i yields the following estimator of the failure rate:

$$\hat{\lambda}_i = \frac{n_i}{\sum_{j=1}^{n_i} t_{ij} + t'_i} = \frac{n_i}{\tau_i} \qquad (2.1)$$

where τ_i represents the total exposure time for secondary reparable i. [Ref. 20:pp.282-296]

The Bayesian approach also assumes that the distribution of the lifetimes for each secondary reparable i is exponential with parameter λ_i; however, it further assumes that the exact value of this parameter is a realization of a random variable. Even though λ_i is not precisely known, it has a prior distribution which is taken here to be a conjugate Gamma distribution with shape parameter $\alpha_i(\alpha_i>0)$ and scale parameter $\beta_i(\beta_i>0)$:

$$f(\lambda_i;\, \alpha_i, \beta_i) = \begin{cases} \dfrac{\beta_i^{\alpha_i}}{\Gamma(\alpha_i)} \lambda_i^{\alpha_i - 1} \exp^{-\beta_i \lambda_i}, & \lambda_i > 0 \\ 0 & , \lambda_i \leq 0 . \end{cases} \qquad (2.2)$$

The prior distribution parameters are estimated using a parametric empirical Bayes (PEB) approach for failure rates as outlined by Gaver and Lehoczky [Ref.21:pp. 220-224]. The

PEB approach uses the entire data set to compute estimates $\hat{\alpha}_i$ and $\hat{\beta}_i$ which are substituted into the applicable formulas. To compute the prior parameters in PEB requires the maximization of the "marginal likelihood function":

$$L_i(\alpha_i, \beta_i : \underline{n}_i, \underline{t}_i) = \prod_{m=1}^{M_i} \int_0^{\infty} \frac{(\lambda_{im} t_{im})^{n_{im}}}{n_{im}!} \exp^{-\lambda_{im} t_{im}} \frac{\beta_i^{\alpha_i}}{\Gamma(\alpha_i)} \lambda_{im}^{\alpha_i - 1} \exp^{-\beta_i \lambda_{im}} d\lambda_{im} \quad (2.3)$$

where,

- m individual source of data for component i (e.g., tank serial #502826)
- M_i total number of data sources for component i
- λ_{im} failure rate for component i of data source m
- t_{im} observed exposure time of component i on data source m
- n_{im} number of failures of component i on data source m
- α_i shape parameter of Gamma prior distribution of failure rate for component i
- β_i scale parameter of Gamma prior distribution of failure rate for component i.

Integration results in the following marginal likelihood function for secondary reparable i:

$$L_i(\alpha_i, \beta_i ; \underline{n}_i, \underline{t}_i) = \prod_{m=1}^{M_i} \frac{\Gamma(n_{im} + \alpha_i)}{n_{im}! \, \Gamma(\alpha_i)} \left(\frac{\beta_i}{t_{im} + \beta_i} \right)^{\alpha_i} \left(\frac{t_{im}}{t_{im} + \beta_i} \right)^{n_{im}} \quad (2.4)$$

which is the product of individual negative binomial distributions with parameters $p = \frac{\beta_i}{t_{im} + \beta_i}$ and $r = \alpha_i$.

The numerical maximization of this function subject to known upper and lower bounds on each α_i and β_i is discussed in Chapter IV. The important result of this maximization is an estimate for the shape ($\hat{\alpha}_i$)and scale ($\hat{\beta}_i$) parameters of the Gamma prior distribution that incorporates the actual observed operational data for each secondary reparable.

Returning to the Bayesian analysis, the n_i observed times between failure and the observed exposure times t'_i during which no failures occurred are all a function of λ_i and give rise to the likelihood function,

$$f\left(t_i \mid \lambda_i\right) = \lambda_i^{n_i} \exp^{-\lambda_i\left(\sum_{j=1}^{n_i} t_{ij} + t'_i\right)} = \lambda_i^{n_i} \exp^{-\lambda_i \tau_i} \qquad (2.5)$$

where τ_i once again represents the total exposure time for secondary reparable i. We are now interested in the updated distribution of λ_i after the times between failure have been observed so as to provide a current estimate of future failure events. This is the <u>posterior distribution</u> and it is proportional to the product of the likelihood function (2.5) and the prior distribution (2.2):

$$f(\lambda_i \mid t_i) \propto f(t_i \mid \lambda_i) f(\lambda_i) \ .$$

Thus,

$$f(\lambda_i \mid t_i) = c_i(\lambda_i^{n_i} \exp^{-\lambda_i \tau_i})(\lambda_i^{\hat{\alpha}-1} \exp^{-\hat{\beta}_i \lambda_i})$$

where c_i is a constant factor which depends on the observed times between failure, but does not depend on λ_i. The value of this constant can be determined using normalization. The resulting posterior distribution is:

$$f(\lambda_i \mid t_i) = f(\lambda_i \mid \text{data}) = \frac{(\tau_i + \hat{\beta}_i)^{n_i + \hat{\alpha}_i}}{\Gamma(n_i + \hat{\alpha}_i)} \lambda_i^{n_i + \hat{\alpha}_i - 1} \exp^{-(\tau_i + \hat{\beta}_i)\lambda_i} \tag{2.6}$$

a Gamma distribution with shape parameter $n_i + \hat{\alpha}_i$ and scale parameter $\tau_i + \hat{\beta}_i$. [Ref. 20:pp.257-280]

The posterior distribution could also have been derived directly using the fact that the family of Gamma distributions serves as a conjugate family of prior distributions for samples from an exponential distribution. Thus, using the theorem for Gamma conjugate priors [Ref. 20:pp. 271-272], the posterior distribution for λ_i given the observed times between failure, is a Gamma distribution with shape parameter $n_i + \alpha_i$ and scale parameter $\tau_i + \beta_i$. This agrees with the result derived in the previous paragraph and indicates that the posterior distribution can be defined knowing only the number of observations and the sum of all the exposure times. This is an important result because it enables the posterior distribution to be updated very easily as new observations are obtained.

Since the posterior distribution of λ_i has been shown to be a Gamma distribution, its expected value is:

$$E[\lambda_i \mid \text{data}] = \frac{n_i + \hat{\alpha}_i}{\tau_i + \hat{\beta}_i} .$$

This expected value also happens to be a "Bayes estimator" of λ_i when a "squared error loss function" is used [Ref. 20:p. 227]. This estimator could also have been used to provide an estimate for each λ_i. However, the primary purpose of the Bayesian analysis was not to provide a point estimate of λ_i, but rather to provide a distribution (i.e., posterior distribution) for each λ_i which could be used to make probabilistic predictions about demand functions involving λ_i. This step is detailed in the next chapter.

III. MODEL FORMULATION AND DESCRIPTION

In this chapter, the two different objective functions considered in this study are defined, derived, and justified. Additionally, each of the three scenarios described in the introduction is formulated as an integer nonlinear program whose objective function is subsequently linearized to create an integer linear problem.

A. MODEL DEVELOPMENT

The following formulation is developed to model the determination of secondary reparable inventory levels for each of the three scenarios described in the introduction. With the exception of the number of constraints, the formulation is the same for each scenario.

INDICES:

$i = 1, \ldots, N$ secondary reparable or component

DATA:

w_i	unit shipping weight of secondary reparable i [lbs]
W	weight capacity of aircraft or ship [lbs]
v_i	unit volume of secondary reparable [cuft]
V	volume capacity of aircraft or ship [cuft]
u_i	maximum allowable number of spares for component i

FUNCTIONS:

$p_i(x_i)$ probability that the number of readiness reportable pacing items (specified by the operational commander) containing secondary reparable i will survive a mission given x_i spares are stocked

DECISION VARIABLE:

x_i Number of spares to stock for secondary reparable i

FORMULATION:

$$\max \prod_{i=1}^{N} p_i(x_i) \qquad (P1)$$

$$\text{s.t.} \quad \sum_{i=1}^{N} w_i x_i \leq W$$

$$\sum_{i=1}^{N} v_i x_i \leq V$$

$$0 \leq x_i \leq u_i \qquad i = 1, \ldots, N$$

$$x_i \quad \text{integer}$$

The objective function is a mathematical expression of the probability that a quantity (specified by the operational commander) of each readiness reportable pacing item in a MAGTF's Table of Equipment will survive a mission of specified duration given a particular stockage policy for secondary reparables. This probability was referred to in the introduction as the probability of mission completion.

In terms of this definition, successful mission completion hinges on the survival of the MAGTF's pacing items. Each pacing item is composed of secondary reparables. The failure of any one secondary reparable will cause the pacing item to be combat deadlined and unable to complete its mission. Thus, each pacing item is viewed as a system of secondary reparables in series. However, in a MAGTF's Table of Equipment, there is not just one of each type of pacing item, but rather a specific quantity of each type necessary to accomplish the assigned mission. This minimum quantity required will not only vary from one mission to another, but also among different MAGTF commanders when confronted with identical missions. Therefore, the different pacing items taken as a whole do not constitute a simple series system in which all components must survive the given mission.

In defining the objective function, two different functional forms were derived and studied. Both rely on the assumptions that secondary reparables fail at constant rates and that the failures of individual secondary reparables are independent of one another. The independence assumption is certainly true for the secondary reparables associated with different types of pacing items; however, it may not necessarily be true for the secondary reparables of a specific pacing item. For example, the failure of M60A1 tank engines is independent of the failure of LVTP7A1

amtrack engines, but the failure of amtrack final drives may not be independent of amtrack transmission failures. These assumptions together with the fact that secondary reparables from one type of pacing item are not interchangeable with those of another (e.g., a tank engine cannot be substituted for an amtrack engine) make it possible to consider only secondary reparables without having to specify the pacing items of which they are components.

The functional form of the first objective function, in which the failure rate of each secondary reparable is assumed to be known, is as follows:

$$p_i(x_i) = \sum_{m=0}^{a_i + x_i - k_i} \frac{(a_i \hat{\lambda}_i t_i)^m}{m!} \exp^{-a_i \hat{\lambda}_i t_i} \qquad (3.1)$$

where,

a_i number of pacing items containing secondary reparable i

k_i minimum number of pacing items containing secondary reparable i required to be operational throughout given mission $(k_i \leq a_i)$

$\hat{\lambda}_i$ M.L.E. of failure rate for secondary reparable i

t_i mission duration during which secondary reparable i must operate.

This function is a direct consequence of the constant failure rate assumption for each secondary reparable. Underlying this constant failure rate is an inherent exponential lifetime distribution which implies that the

number of failures defines a Poisson (arrival) process with exponential interarrival times. The number of failures of secondary reparable i to occur in any fixed interval of time, such as a mission of duration t_i, will have a Poisson distribution with mean $\lambda_i t_i$. However, since there are a total of a_i pacing items containing secondary reparable i and operating during this mission, the total number of failures will be the sum of a_i independent, Poisson random variables (each with a mean $_i t_i$) which is itself a Poisson random variable with a mean $a_i \lambda_i t_i$. This is really only an approximation since not all a_i pacing items may be required during the mission. In which case the overall failure rate would be smaller implying that we are probably overestimating the number of spares required. On the other hand, it can be argued that with fewer than a_i items the overall failure rate of the operational items may increase since the same mission must be accomplished with fewer assets. In any case, this can be viewed as a mild, worst case approximation.

Therefore, equation (3.1) represents the probability that there are less than or equal to some specified number of failures of secondary reparable i during a mission of duration t_i. This specified number is the sum of the number of spares, x_i, and the difference between the number of pacing items containing secondary reparable i (a_i) and the

required minimum number of pacing items containing secondary reparable i (k_i). As an example, consider the situation where the Table of Equipment of some MAGTF consists of five tanks each containing one engine (a_i=5) and assume that the MAGTF commander decides that to accomplish the assigned mission he must have a minimum of four tanks operational at all times (k_i=4). Then, assuming one spare tank engine (x_i=1) is stocked, there can be no more than two engine failures ($a_i+x_i-k_i$=5+1-4=2) to adhere to the commander's guidance. It can be shown that this probability is equal to the probability that the quantity of each type of pacing item (or equivalently all its component secondary reparables) remains above a specified level (k_i), because the following relation must always hold true:

{number of failures of secondary reparable i} +

{number of secondary reparables i operational and in inventory} =

{total number of secondary reparables i, (a_i + x_i)}

Thus,

P({number of secondary reparables i operational and in inventory at t}$\geq k_i$) =

P({a_i+x_i} - {number of failures of secondary reparable i in t}$\geq k_i$)=

P({number of failures of secondary reparable i in t} $\leq a_i+x_i-k_i$).

Therefore, the probability of mission completion for the entire MAGTF is the product of the probabilities that the

individual secondary reparables exceed a level, k_i, given x_i spares (or equivalently experience at most $a_i+x_i-k_i$ failures). So, the maximum number of operational pacing items of a specific type at any given moment will be equal to the lowest level of its component secondary reparables. It is this minimum level that must exceed the specified level k_i, since if the minimum level exceeds k_i, all levels will exceed k_i. Now, by the assumption that the secondary reparable failures are independent combined with the fact that if one type of pacing item cannot complete the mission the entire MAGTF will not be able to complete the mission, the probability that the minimum level exceeds the specified level (i.e., $p_i(x_i)$) can be expressed as,

$$p_1(x_1) \cdot p_2(x_2) \cdot \ldots \cdot p_N(x_N) = \prod_{i=1}^{N} p_i(x_i) \qquad (3.2)$$

which accounts for the functional form of the objective function.

Similar to the first objective function, the second objective function also takes advantage of the assumptions of constant failure rates for each secondary reparable and independence among the failures of different secondary reparables; however, it incorporates a more realistic approach as to the nature of the failure rates. In the previous objective function, the failure rate λ_i, of each secondary reparable was assumed to be known (actually an

estimate computed using the maximum likelihood estimator of the failure rate must realistically be used). For this second objective function, a Bayesian approach is taken in which the exact value of λ_i is explicitly recognized to be unknown. The observed data (i.e., times to failure) are used to update the prior information concerning each failure rate to form a posterior distribution as discussed in the chapter on data analysis. This posterior distribution is necessary to derive the predictive distribution and the functional form of $p_i(x_i)$, since we are interested in computing the probabilty distribution of the number of failures of secondary reparable i which occur during some mission of length t_i. By analogy to the previous derivation of $p_i(x_i)$, it is desired to consider this number of failures because we want this quantity to be less than some specified level (i.e., $a_i+x_i-k_i$) so as to insure the entire MAGTF completes the assigned mission. Assuming that the failure rate λ_i is given, the total number of failures of a_i secondary reparables of type i in a mission of length t_i will have a Poisson distribution with approximate mean $a_i \lambda_i t_i$,

$$P(\text{Exactly } f_i \text{ failures in } (0,t_i] \mid \lambda_i) = \frac{(a_i \lambda_i t_i)^{f_i}}{f_i!} \exp^{-a_i \lambda_i t_i}$$

But λ_i is not precisely known and has the following posterior density (as derived in the data analysis chapter)

given the observed times to failure and prior information:

$$f(\lambda_i \mid \text{data}) = \frac{(\tau_i + \hat{\beta}_i)^{n_i + \hat{\alpha}_i}}{\Gamma(n_i + \hat{\alpha}_i)} \lambda_i^{n_i + \hat{\alpha}_i - 1} \exp^{-\lambda_i(\tau_i + \hat{\beta}_i)}$$

where,

τ_i total exposure time for secondary reparable i

n_i number of observations for secondary reparable i

$\hat{\alpha}_i$ empirical Bayes estimate of shape parameter for Gamma ($\lambda_i ; \alpha_i , \beta_i$) prior distribution

$\hat{\beta}_i$ empirical Bayes estimate of scale parameter for Gamma ($\lambda_i ; \alpha_i , \beta_i$) prior distribution.

This can be rewritten as,

$$f(\lambda_i \mid \text{data}) = \frac{(\lambda_i(\tau_i + \hat{\beta}_i))^{n_i + \hat{\alpha}_i - 1} (\tau_i + \hat{\beta}_i) \exp^{-\lambda_i(\tau_i + \hat{\beta}_i)}}{\Gamma(n_i + \hat{\alpha}_i)} .$$

Now, to calculate the predictive distribution,

$$P(\text{Exactly } f_i \text{ failures in } (0,t_i]) = \int_0^\infty P(\text{Exactly } f_i \text{ failures in } (0,t_i] \mid \lambda_i)\, f(\lambda_i \mid \text{data}) d\lambda_i$$

which is upon substitution,

$$\int_0^\infty \left(\frac{(a_i \lambda_i t_i)^{f_i}}{f_i!} \exp^{-a_i \lambda_i t_i} \right) \left(\frac{(\tau_i + \hat{\beta}_i)^{n_i + \hat{\alpha}_i}}{\Gamma(n_i + \hat{\alpha}_i)} \lambda_i^{n_i + \hat{\alpha}_i - 1} \exp^{-\lambda_i(\tau_i + \hat{\beta}_i)} \right) d\lambda_i .$$

Evaluation of the integral yields:

$$P(\text{Exactly } f_i \text{ failures in } (0,t_i]) = \frac{\Gamma(n_i + \hat{\alpha}_i + f_i)}{f_i!\, \Gamma(n_i + \hat{\alpha}_i)} \left(\frac{\tau_i + \hat{\beta}_i}{a_i t_i + \tau_i + \hat{\beta}_i} \right)^{n_i + \hat{\alpha}_i} \left(\frac{a_i t_i}{a_i t_i + \tau_i + \hat{\beta}_i} \right)^{f_i} .$$

This is a <u>negative binomial distribution</u> with parameters

$$p = \frac{\tau_i + \hat{\beta}_i}{a_i t_i + \tau_i + \hat{\beta}_i} \quad \text{and} \quad r = n_i + \hat{\alpha}_i .$$

Even though $p_i(x_i)$ is defined as the probability that the stockage of x_i spares for secondary reparable i allows the pacing item of which it is a component to exceed some minimum level (k_i), it has been shown that this is equivalent to the probability that the number of failures of secondary reparable i during mission t_i is less than or equal to $a_i+x_i-k_i$ (i.e., P(number of failures of secondary reparable i in $(0,t_i] \leq a_i+x_i-k_i$)). Therefore, the second functional form of $p_i(x_i)$ is:

$$p_i(x_i) = \sum_{m=0}^{a_i+x_i-k_i} \frac{\Gamma(n_i+\hat{\alpha}_i+m)}{m!\,\Gamma(n_i+\hat{\alpha}_i)} \left(\frac{\tau_i+\hat{\beta}_i}{a_it_i+\tau_i+\hat{\beta}_i}\right)^{n_i+\hat{\alpha}_i} \left(\frac{a_it_i}{a_it_i+\tau_i+\hat{\beta}_i}\right)^{m} \qquad (3.3)$$

where,

- a_i number of pacing items containing secondary reparable i
- k_i minimum number of pacing items containing secondary reparable i required to be operational throughout given mission ($k_i \leq a_i$)
- n_i number of observations (data points) of times between failure for secondary reparable i
- τ_i total observed exposure time for secondary reparable i
- t_i mission duration during which secondary reparable i must operate
- $\hat{\alpha}_i$ empirical Bayes estimate of shape parameter for Gamma ($\lambda_i;\alpha_i,\beta_i$) prior distribution
- $\hat{\beta}_i$ empirical Bayes estimate of scale parameter for Gamma ($\lambda_i;\alpha_i,\beta_i$) prior distribution.

The reasoning behind the functional form of equation (3.2) developed for the first $p_i(x_i)$ still holds true for the

newly derived $p_i(x_i)$; therefore, the functional form of this objective function is still the product of the individual $p_i(x_i)$'s.

The linear weight capacity restricts the total shipping weight (i.e., secondary reparable plus container) for all secondary reparables stocked to be less than or equal to the weight capacity of the ship or airplane. Similarly, the linear constraint on volume or space capacity requires that all secondary reparables stocked not exceed the volume available. The final constraint in P1 indicates that there is an upper bound on the number of spares for each secondary reparable that can be stocked. This is a very real constraint because there are only a finite number of each type of secondary reparable at the wholesale inventory level from which the retail inventories used to support all exercises, deployments, and contingencies can be established. Thus, a combat service support planner must consider the requirements of the force as a whole so as not to commit too many secondary reparables to any one exercise.

The number of constraints is dependent on the scenario. Any scenario that involves the airlift of the ground combat element and its supporting forces into the exercise or operation area, such as the short-term exercises described in the introduction, will have only one constraint. This is

the constraint on the allowable cargo load (ACL). Allowable cargo load is:

> The initial limiting factor in determining airlift requirements. . . . This is a weight limiting factor expressed in terms of STON [short tons] which cannot be exceeded in planning aircraft loads. The ACL is established by the Air Force for each type aircraft for a specific mission. [Ref. 22:pp. 4-9]

Since the formulation of problem P1 now reduces to just one constraint, it is recognizable as a nonlinear knapsack problem due to the nonlinear objective function. The individual secondary reparables represent the items or commodities that must fit into a knapsack while the ACL constraint corresponds to the knapsack's weight or volume capacity.

The other two scenarios described in the introduction both involve two constraints. These two constraints represent the weight and volume capacities of the shipping assigned to perform the sealift. In the case of the MAUs deploying to the Mediterranean Sea, the secondary reparables are normally stored aboard one of the ships of the amphibious task force such as a LHA, LPH, LKA, LSD, or LST. Thus, the constraints reflect the weight and volume capacity of the ship's cargo hold or compartment which has been designated for secondary reparable storage. On the other hand, in considering secondary reparable support of the MPF MAB, the secondary reparables are spread-loaded among several of the MPF squadron ships; therefore, the weight and

volume constraints are actually the combined weight and volume capacities of the cargo holds of the specified MPF squadron ships. Since both of these sealift scenarios involve two contraints, problem P1 can no longer be regarded as a nonlinear knapsack problem, but it is still an integer nonlinear programming problem.

Regardless of the number of constraints or whether equation (3.1) or (3.3) is used as the functional form for $p_i(x_i)$, the objective function is a complex, nonlinear function of the decision variable. For this reason, problem P1 will be transformed into a binary linear programming problem. The following section explains this transformation and the new formulation.

B. TRANSFORMATION TO A BINARY LINEAR PROGRAM

It can be seen from the final constraint of problem P1 that the decision variable, x_i, is constrained to a set of discrete values. Therefore the objective function will first be separated using the properties of logarithms and then the transformation technique for converting discrete variables to binary variables will be used to reformulate the problem [Ref. 23:p. 12]. Taking the natural logarithm of the objective function yields:

$$\ln\left(\prod_{i=1}^{N} p_i(x_i)\right) = \sum_{i=1}^{N} \ln p_i(x_i) .$$

Now, the final constraint in P1 can be expressed as,

$$x_i \in S_i = \{s_{i0}, s_{i1}, \dots, s_{iu_i}\} = \{0,1,\dots,u_i\}$$

and this is equivalent to the constraint set [Ref. 23:p. 12]:

$$x_i = \sum_{j=0}^{u_i} s_{ij} x_{ij}$$

$$\sum_{j=0}^{u_i} x_{ij} = 1$$

$$x_{ij} \in \{0,1\} \quad , \quad j = 0,1,\dots,u_i \ .$$

Applying the separation technique and transformation to problem P1 results in the new formulation:

INDICES:

$i = 1,\dots,N$	secondary reparable or component
$j = 0,\dots,u_i$	number of secondary reparables i

DATA:

f_{ij}	$\ln(p_i(j))$
w_{ij}	total shipping weight for j spares of type i [lbs]
W	weight capacity of airplane or ship [lbs]
v_{ij}	total volume of j spare of type i[cuft]
V	volume capacity of airplane or ship [cuft]
u_i	maximum allowable number of spares for component i

DECISION VARIABLE:

$$x_{ij} = \begin{cases} 1 & \text{if j spares of type i are selected} \\ 0 & \text{otherwise} \end{cases}$$

FORMULATION:

$$\max \quad \sum_{i=1}^{N} \sum_{j=0}^{u_i} f_{ij} x_{ij} \qquad (P2)$$

$$s.t. \quad \sum_{i=1}^{N} \sum_{j=0}^{u_i} w_{ij} x_{ij} \leq W$$

$$\sum_{i=1}^{N} \sum_{j=0}^{u_i} v_{ij} x_{ij} \leq V$$

$$\sum_{j=0}^{u_i} x_{ij} = 1 \qquad \forall i$$

$$x_{ij} \in \{0,1\}$$

Assuming known failure rates, the coefficients, f_{ij}, in the objective function are:

$$f_{ij} = \ln\left[\sum_{m=0}^{a_i + j - k_i} \frac{(a_i \hat{\lambda}_i t_i)^m}{m!} \quad \exp^{-a_i \hat{\lambda}_i t_i} \right] \qquad (3.4)$$

where,

a_i number of pacing items containing secondary reparable i

k_i minimum number of pacing items containing secondary reparable i required to be operational throughout given mission ($k_i \leq a_i$)

$\hat{\lambda}_i$ M.L.E. of failure rate for secondary reparable i

t_i mission duration during which secondary reparable i must operate.

If perfect knowledge about the failure rates is not assumed and the Bayesian approach is adopted the coefficients are:

$$f_{ij} = \ln\left[\sum_{m=0}^{a_i + j - k_i} \frac{\Gamma(n_i + \hat{\alpha}_i + m)}{m!\,\Gamma(n_i + \hat{\alpha}_i)} \left(\frac{\tau_i + \hat{\beta}_i}{a_i t_i + \tau_i + \hat{\beta}_i} \right)^{n_i + \hat{\alpha}_i} \left(\frac{a_i t_i}{a_i t_i + \tau_i + \hat{\beta}_i} \right)^{m} \right] \tag{3.5}$$

where,

- a_i number of pacing items containing secondary reparable i
- k_i minimum number of pacing items containing secondary reparable i required to be operational throughout given mission ($k_i \le a_i$)
- n_i number of observations (data points) of times between failure for secondary reparable i
- τ_i total observed exposure time for secondary reparable i
- t_i mission duration during which secondary reparable i must operate
- $\hat{\alpha}_i$ empirical Bayes estimate of shape parameter for Gamma (λ_i; α_i, β_i) prior distribution
- $\hat{\beta}_i$ empirical Bayes estimate of scale parameter for Gamma (λ_i; α_i, β_i) prior distribution.

It is of interest to note that for situations in which the airlift model is applicable, problem P2 is just a special case of the generalized assignment problem. In the formulation of the generalized assignment problem [Ref. 24:pp.345-346], each agent (or man) can perform or be assigned more than one task (or job) provided that the resource available to the agent is not exceeded; whereas, in the classical assignment problem [Ref. 23:pp. 61-62], each agent can be assigned to only one task. In both formulations, each task must be assigned to exactly one of

the agents. The airlift model can be interpreted as a special case of this generalized assignment problem in which one agent must perform all N tasks, however, he has $(u_i + 1)$ alternative ways to accomplish each task.

Problem P2 is equivalent to P1 and even though the number of variables and constraints have increased, it is the preferred formulation because it eliminates the complex nonlinear objective function of P1. The coefficients f_{ij} can be directly computed using recursive formulas for the Poisson or negative binomial cumulative distribution functions; whereas, $p_i(x_i)$ could not since it was itself a function of the decision variable. Additionally, because of the assumption of the independence of failures, other failure distributions can be readily introduced into this linear formulation. Furthermore, the formulation is general enough so that additional constraints can be incorporated as required and as will be seen in the following chapter, it lends itself easily to programming in an algebraic modeling language.

IV. SOLUTION METHODOLOGY AND IMPLEMENTATION

The purpose of this study is to determine an optimal stockage level of spare secondary reparables for each of the three different scenarios described in the introduction. Of course "optimality" is with reference to the failure model adopted and the relevance of the data used to estimate its parameters. The determination of the stockage levels is the subject of this chapter. The binary linear nature of the formulation presented in the preceding chapter suggests that several different methods of solution are available to determine the optimal stockage policy for a given scenario. Each alternative method will be discussed and justification for the method chosen is provided in addition to its implementation.

A. SOLUTION APPROACHES AND METHODOLOGY

The mode of deployment in a particular scenario determines the number of constraints. The initial solution approach to the scenario in which the airlift model was applicable utilized the technique of Lagrangian relaxation. Although it is standard procedure to establish bounds for integer programs (such as problem P2) using a linear programming (LP) relaxation, it was initially desired to avoid dependency on commercial LP solvers not locally

available to the combat service support planners of the Second Force Service Support Group at Camp Lejeune, North Carolina. Therefore, Lagrangian relaxation was adopted because it lends itself nicely to programming in readily available FORTRAN 77. It must be emphasized that the structure of problem P2 did not demand that this specific solution technique be applied. In fact, the LP relaxation of problem P2 could have been solved using an LP solver. Thus, Lagrangian relaxation was utilized not out of necessity, but primarily because it offered a realistic alternative to the use of a commercial LP optimizer.

Briefly stated, the technique of Lagrangian relaxation, as outlined by Fisher [Ref. 25:pp. 10-21], involves moving complicating constraints into the objective function using the product of the Lagrangian multiplier and the constraint violation as a penalty term. In particular, examination of problem P2 in the context of the airlift scenario results in the following binary linear formulation (actually a generalized assignment formulation):

INDICES:

$i = 1, \ldots, N$ secondary reparables or components

$j = 0, \ldots, u_i$ number of secondary reparables i

DATA:

f_{ij}	$\ln(p_i(j))$
w_{ij}	total shipping weight for j spares of type i [lbs}
ACL	allowable cargo load of airplane [lbs]
u_i	maximum allowable number of spares for component i

DECISION VARIABLE:

$$x_{ij} = \begin{cases} 1 & \text{if j spares of type i are selected} \\ 0 & \text{otherwise} \end{cases}$$

FORMULATION:

$$\max \sum_{i=1}^{N} \sum_{j=0}^{u_i} f_{ij} x_{ij} \qquad (P3)$$

$$s.t. \sum_{i=1}^{N} \sum_{j=0}^{u_i} w_{ij} x_{ij} \leq ACL$$

$$\sum_{j=0}^{u_i} x_{ij} = 1 \qquad \forall i$$

$$x_{ij} \in \{0,1\}$$

The only complicating constraint is the allowable cargo load (ACL) constraint. The relaxed formulation with _scalar_ Lagrangian multiplier, λ , consists of N separable, _multiple choice_ problems which are easy to solve. On the other hand, for those situations in which both the weight and volume constraints of problem P2 must be considered (i.e., Mediterranean MAU or MPF MAB scenarios), the Lagrangian multiplier, λ , is now vector-valued (i.e., $\lambda = (\lambda_1, \lambda_2)$ since two constraints must be relaxed, and this is no longer a

trivial problem to solve. The primary reason for this complication is that the solution approach now involves a two-dimensional search which is more difficult to program than the simple bisection search associated with the single, scalar Lagrangian multiplier. The problem is not impossible to solve and methods exist, such as a coordinate search method suggested by DeWolfe, Stevens, and Wood [Ref.26: pp.8-10], which appears to offer good results for this type of problem.

As additional constraints (such as budget, holding (storage) cost, or shortage (stockout) cost constraints) are added, the programming difficulty associated with the search procedure of the Lagrangian function becomes prohibitive. Therefore, to accommodate the possible inclusion of additional constraints in the formulation of problem P2, it was decided to utilize the Generalized Algebraic Modeling System (GAMS) in conjunction with the Zero/One Optimization Methods (ZOOM) mixed integer program solver. The use of this optimization package not only avoids the restrictive programming considerations discussed above, but it can also incorporate the PEB as an initial step, prior to solution of P2 or P3.

The approach to the PEB is to maximize the marginal likelihood function associated with each secondary reparable i with respect to α_i and β_i. This marginal likelihood

function is a nonlinear function of the two decision variables. The formulation for this maximization problem is:

INDICES:

$i = 1, \ldots, N$ secondary reparable or component

$m = 1, \ldots, M_i$ source of data (e.g., tank serial number 508282)

DATA:

n_{im} number of observations of component i failures for data source m

t_{im} total observed exposure time for component i of data source m.

$\overline{\alpha}_i$ upper bound on the shape parameter of the Gamma prior distribution for component i.

$\overline{\beta}_i$ upper bound on the scale parameter of the Gamma prior distribution for component i.

DECISION VARIABLES:

$\hat{\alpha}_i$ shape parameter for Gamma prior distribution associated with compenent i.

$\hat{\beta}_i$ scale parameter for Gamma prior distribution associated with component i.

FORMULATION:

$$\max \prod_{m=1}^{M_i} \frac{\Gamma(n_{im} + \hat{\alpha}_i)}{n_{im}!\,\Gamma(\hat{\alpha}_i)} \left(\frac{\hat{\beta}_i}{t_{im} + \hat{\beta}_i} \right)^{\hat{\alpha}_i} \left(\frac{t_{im}}{t_{im} + \hat{\beta}_i} \right)^{n_{im}} \qquad (P4)$$

$$s.t. \quad 0 < \hat{\alpha}_i < \overline{\alpha}_i$$

$$0 < \hat{\beta}_i < \overline{\beta}_i$$

Standard maximization by taking partial derivatives with respect to α_i and β_i does not yield a satisfactory closed

form solution which can be practically implemented. Thus, this nonlinear maximization problem is also solved using GAMS in conjunction with the nonlinear program solver, MINOS [Ref. 27]. These values are then incorporated into the optimization of problem P2. In summary, under the current solution approach, the entire model can be coded for computer processing on either a mainframe system or personal computer and solved using a single modeling system (i.e., GAMS) and its compatible solvers. This approach is sufficiently flexible to incorporate additional constraints or decision variables.

B. IMPLEMENTATION

Since the decision was made to determine the optimal stockage policy for all scenarios using GAMS, no specifics of the original solution approach involving Lagrangian relaxation will be mentioned other than the fact that it is implemented by means of a FORTRAN 77 computer program. The FORTRAN program is based upon a similar one developed by DeWolfe [Ref. 17:pp. 23-24]. A copy of the source code developed for this study is provided in Appendix A and examples of its two required input files and of the single output file are given in Appendices B and C, respectively.

The solution approach adopted centers around the coding of problems P2 and P4 for computer processing using the GAMS language. The reader is referred to the excellent tutorial

by Rosenthal [Ref. 28] for the specifics of this versatile mathematical modeling language. Upon coding, the problems are solved using the GAMS compatible solvers of MINOS and ZOOM for nonlinear programs and mixed integer programs respectively. Specifically, the source code provided in Appendix D incorporates the successive solve feature of GAMS to first solve the nonlinear maximization problem represented by problem P4. The results of this maximization provide the estimates of the shape ($\hat{\alpha}_i$) and scale ($\hat{\beta}_i$) parameters of the prior distributions which are required before the second binary linear maximization (P2) can be performed. The final result is the optimal stockage level of spare secondary reparables for the given scenario. Even though the source code in Appendix D is specific to the airlift model (P3), GAMS makes it very easy to incorporate additional constraints. For example, if a volume constraint is also applicable, a single GAMS PARAMETER statement to accomodate the volume data and EQUATION statement to reflect the volume constraint are the only additions necessary.

The source code provided is compatible with either mainframe or personal computer usage. When run on the IBM 3033AP mainframe computer, GAMS version 2.05 was utilized in conjunction with MINOS Version 5.1 and ZOOM version 2.1 and on the Zenith Z-248 personal computer (equipped with 80287-8 math coprocessor and 640 Kbyte memory expansion board), PC

GAMS Version 2.04 was utilized in conjunction with MINOS Version 5.0 and ZOOM Version 2.1.

V. RESULTS

This chapter summarizes results obtained using actual operational maintenance data to determine an optimal level of spare secondary reparables for a Marine Amphibious Unit (MAU) in a realistic exercise scenario. Verification of the model is accomplished using simulated failure data. Estimation of the parameters of the prior distribution by an analysis of the data is discussed and demonstrated; this realizes computationally the PEB procedure. Finally, computational experience for both the mainframe and Zenith-248 personal computer is briefly discussed.

A. MODEL TESTING

The data used in the development and testing of all models was provided by the Operations Section of the Supported Activities Supply System (SASSY) Management Unit in coordination with the Maintenance Information Systems Management Office of the Second Force Service Support Group at Camp LeJeune, North Carolina. This data was extracted from the ERO History File of the Marine Corps Integrated Maintenance Management System and analyzed according to the procedure presented in Chapter II. This analysis provided the basic parameters for equation (3.5) (or (3.4)) from

which the objective function coefficients were generated using GAMS.

The airlift model (P3) was used to determine the optimal stockage policy for twenty-five different secondary reparables associated with the pacing items of a Marine Amphibious Unit participating in a realistic short-term exercise. This problem generates 190 binary variables and 26 constraints. The recommended stockage level for the secondary reparables associated with the three pacing items (i.e., M60A1 Tank, LTP7A1 Amtrack, and M101A1 105mm Howitzer) in this exercise is presented in Table 2. Table 2 displays the levels computed using both the GAMS/ MINOS/ZOOM package and the Lagrangian relaxation technique.

For this exercise scenario 99.89% of the allowable cargo load was utilized and the objective function value was within 0.07% of the upper bound established during the branch and bound procedure used by the ZOOM solver. The probability of mission completion given the stockage level was 0.4149 and this highlights one of the advantages associated with using this model. This low probability implies that the allowable cargo load must be increased. The only way to accomplish this is to increase the number of cargo planes assigned to this exercise. This provides Marine Corps planners with a tangible basis on which to

TABLE 2

MAU AIRLIFT SCENARIO SPARE STOCKAGE LEVELS

NSN	END ITEM	SPARE STOCKAGE LEVEL GAMS/ ZOOM	LAGRANGIAN RELAXATION
2815001245387	M60A1	2	2
2520002241867	M60A1	3	3
2815002395819	M60A1	2	2
2520001549632	M60A1	2	2
2530000886657	M60A1	2	3
2520001184942	M60A1	2	2
2920010135802	M60A1	1	1
1010010703803	M60A1	1	1
1025019388105	M60A1	4	5
1010010720397	M60A1	1	1
1025010708993	M60A1	1	1
2815004303480	LVTP7A1	3	4
2520003973384	LVTP7A1	5	5
2520001443385	LVTP7A1	4	4
5805014593410	LVTP7A1	4	5
2520008949532	LVTP7A1	2	2
2910011714636	LVTP7A1	2	3
2530011509757	LVTP7A1	2	2
5865012207848	LVTP7A1	3	3
4320012035660	LVTP7A1	2	2
2920002317276	LVTP7A1	1	1
2530000886650	LVTP7A1	2	2
1005011855059	M101A1	5	4
1005011086434	M101A1	5	5
1005011457709	M101A1	5	5

justify their requests for additional airlift assets from the Military Airlift Command of the U.S. Air Force. Table 3 summarizes the results for both solution approaches.

TABLE 3

SUMMARY OF RESULTS FOR MAU AIRLIFT SCENARIO

Solution Technique	Objective Function Value	Probability of Mission Completion	Percentage of ACL Used	Deviation from Optimality
GAMS/MINOS/ ZOOM	-0.8797	0.4149	99.89%	0.07%
Lagrangian Relaxation (w/Heuristic)	-0.8820	0.4139	100%	0.47%

B. MODEL VERIFICATION

Verification of the modeling approach was conducted by using the model to verify the fact that the negative binomial predictive function reduces to the Poisson-Gamma function as the number of failures observed (and accordingly the total exposure time) approaches infinity. Thus, the Poisson function provides the limiting case or best possible stockage level attainable assuming that the failure rates for each secondary reparable are precisely known.

To accomplish this verification, the MAU airlift model was run using the Poisson objective function (3.4) with fixed failure rates. The resulting stockage levels represent the best stockage policy that can be achieved.

Then, using those failure rates, twenty-five different sets of simulated failure data were generated for each of 10, 25, 50, 100, 200 and 500 failures. These simulated failure times were randomly generated using the probability integral transform to obtain exponential failure times from uniform (0,1) deviates provided by the LLRANDOMII Random Number Generation Package [Ref. 29]. The Bayesian analysis of this data was performed resulting in the negative binomial predictive distribution and the objective function, equation (3.5). A noninformative Gamma prior distribution with $\alpha=1$ and $\beta=1$ was employed in this process. Stockage levels for this method are displayed in Table 4. Note that about 100 failure observations are needed before the stockage levels approach those computed using the actual failure rates (i.e., the optimal stockage policy given perfect information). Figure 1 is a plot of the interquartile range (IQR) of the probabiltiy of mission completion associated with each failure sample size. This graph demonstrates that as the number of observed failures increases, the variability (represented by the IQR) in the probability of mission completion decreases.

In addition to the above verification, the fact that the model and solution approach responded predictably to a wide range of different failure observations and exposure times indicates that the model and approach tend to be robust.

TABLE 4

MAU AIRLIFT SCENARIO SPARE STOCKAGE LEVELS

SPARE STOCKAGE LEVEL

	NEGATIVE BINOMIAL						
NSN	10	25	50	100	200	500	POISSON
2815001245387	3	3	3	2	2	2	2
2520002241867	4	4	4	3	3	3	3
2815002395819	2	2	2	2	2	2	2
2520001549632	2	2	2	3	3	3	3
2530000886657	1	1	1	2	2	2	2
2520001184942	1	1	1	2	2	1	1
2920010135802	1	1	1	1	1	1	1
1010010703803	0	0	1	1	1	1	1
1025019388105	4	4	4	4	4	4	4
1010010720397	1	1	1	1	1	1	1
1025010708993	1	0	0	1	1	1	1
2815004303480	5	4	4	4	4	4	4
2520003973384	5	5	5	5	5	5	5
2520001443385	3	4	4	4	4	4	4
5805014593410	5	5	5	5	5	5	5
2520008949532	2	2	2	2	2	2	2
2910011714636	1	2	2	2	2	2	2
2530011509757	2	2	2	2	2	2	2
5865012207848	3	2	2	3	3	3	3
4320012035660	3	2	2	2	2	2	2
2920002317276	1	1	1	1	1	1	1
2530000886650	1	1	1	1	1	2	2
1005011855059	4	4	3	4	4	4	4
1005011086434	5	5	5	5	5	5	5
1005011457709	5	5	5	5	5	5	5

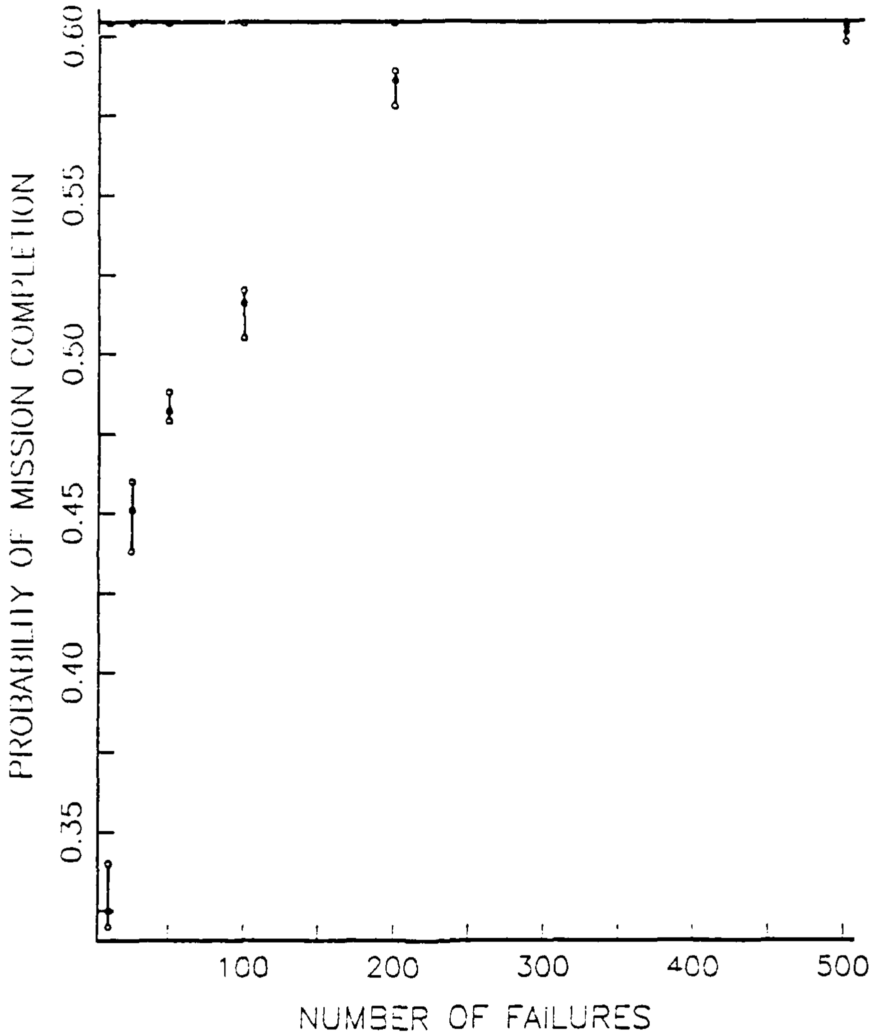

Figure 1. Variability in P(Mission Completion) with Number of Observed Failures

C. BAYESIAN AND EMPIRICAL BAYES ANALYSIS

Bayesian analysis and particularly PEB suggest themselves when the failure rates of different units (e.g., tank engines on different tanks or amtrack transmissions on different amtracks) are sufficiently alike, but individual experience is small, so that "borrowing strength" by a suitable pooling process can improve individual estimates. In this study, the failure rate associated with each secondary reparable (λ_i) was assumed to follow a Gamma prior distribution. This prior distribution represents the probability that the unknown value of the failure rate lies in various regions of the parameter space [Ref. 20:p. 260]. In other words, incorporating the prior distribution enables a tighter bound on the unknown value of the failure rate to be obtained than would be possible by using the data pertaining to that rate alone. The primary disadvantage of the ordinary or classical Bayesian approach is that the prior distribution's parameters (i.e., α_i and β_i) are assumed to be known. This deficiency is overcome in this study by employing a PEB approach, in which the entire data set ($(n_{im}, t_{im}), m = 1, \ldots, M_i$) associated with each secondary reparable i is used to estimate the parameters of the prior distribution. These estimates are then used in the standard Bayesian analysis to compute the posterior and predictive distributions. The results are unavoidably influenced by

the particular choice of prior used (here a Gamma). Sentitivity tests have not been conducted, but indication is that the procedure is rather insensitive.

A specific indication of this insensitivity is seen by comparing the airlift model stockage levels obtained when minimal prior information is included to the levels obtained when the empirical Bayes estimates of the prior distribution are included. The results of this comparison are summarized in Table 5.

The levels in the column labeled "NONINFORMATIVE PRIOR" were obtained by running the airlift model with the randomly generated failure times (described in the previous section) associated with the number of failures sample size of ten. A "diffuse" or noninformative prior distribution (i.e., $\alpha_i = \beta_i = 1$) was used. A diffuse prior distribution is "informationless in a relative sense and means only that it is diffuse relative to the sample information".[Ref. 30:p. 198] The same airlift model was then run using empirical Bayes estimates for the prior parameters and these levels appear in the column labeled "INFORMATIVE PRIOR". These levels are only slightly closer to the Poisson levels than

TABLE 5

EMPIRICAL BAYES ESTIMATION

STOCKAGE LEVELS

NSN	NEGATIVE BINOMIAL NONINFORMATIVE PRIOR	NEGATIVE BINOMIAL INFORMATIVE PRIOR	POISSON
2815001245387	3	2	2
2520002241867	4	4	3
2815002395819	2	2	2
2520001549632	2	2	3
2530000886657	1	1	2
2520001184942	1	1	1
2920010135802	1	1	1
1010010703803	0	0	1
1025019388105	4	4	4
1010010720397	1	1	1
1025010708993	1	1	1
2815004303480	5	5	4
2520003973384	5	5	5
2520001443385	3	4	4
5805014593410	5	5	5
2520008949532	2	2	2
2910011714636	1	1	2
2530011509757	2	2	2
5865012207848	3	3	3
4320012035660	3	3	2
2920002317276	1	1	1
2530000886650	1	1	2
1005011855059	4	4	4
1005011086434	5	5	5
1005011457709	5	5	5

are those computed when minimal prior information is considered.

Even though these stockage levels are only slightly closer to the Poisson levels for this specific instance, by simply using the available data for 10 failures to compute empirical Bayes estimates of the prior distribution parameters, spare stockage levels comparable to those obtained in the previous section using failure sample sizes of between 50 to 100 can be achieved. This is particularly significant since we have no control over the number of failures observed for each secondary reparable or their exposure times and would not usually observe this number of failures (i.e., 50-100) during the reporting period covered by the ERO History File. Also, since operational reliability data may not be available for the exact location where the MAGTF is deploying, pooling the data from sources which have operated in similar environments has the advantage of providing more realistic failures rates than if this prior information were not considered.

D. COMPUTATIONAL EXPERIENCE

So as to keep the implementation and solution of the models developed in this study as general and flexible as possible existing software packages (i.e., STATGRAPHICS, GAMS, etc.) were utilized. The goal was to obtain solutions in a relatively user-friendly personal computer environment

requiring a minimum of operator interaction. Therefore, computational efficiency in terms of computer run time (or algorithmic complexity) was not of primary concern and is included here solely for completeness. No attempt was made to employ special algorithms that exploit problem structure. This is not to downplay the importance of this factor, but rather to emphasize that during this study the primary concern was to obtain solutions to real-world problems as easily and efficiently as possible with a minimum of programming effort.

Running time for the airlift model on the IBM 3033AP using the GAMS/MINOS/ZOOM package was 3.9 seconds in contrast to the 176 seconds obtained when run on the Zenith Z-248 PC. It is possible on the IBM 3033AP to combine the PEB calculations with the maximization of problem P3 by using the successive solve feature of GAMS; however, this cannot be done on the Zenith Z-248 PC since internal parameters in the PC version of GAMS are exceeded. Therefore, each of the empirical Bayes maximizations must be performed separately when using this PC. The resulting estimates have to be manually input into the appropriate GAMS PARAMETER statements before maximization of problem P3 can be accomplished.

Even though the Lagrangian relaxation procedure with heuristic was not adopted as the solution technique for the

reasons discussed in Chapter IV, it provides much faster solution times to the airlift model (P3) on both the IBM 3033AP and Zenith Z-248 PC. The Lagrangian procedure, coded in FORTRAN 77, provides a running time of 0.32 seconds on the IBM 3033AP and about 30 seconds on the Zenith Z-248 PC where it was coded using Microsoft FORTRAN (Version 3.2).

Finally, comparison of the stockage levels obtained using GAMS/MINOS/ZOOM with those obtained using LP relaxation for each of the 150 simulated failure time cases revealed that in every case the LP relaxation provided identical stockage levels with the exception that it fractionated for exactly one of the twenty-five secondary reparables. So, if a general policy was adopted to round down to the lower of the fractionated stockage levels (so as not to violate the ACL contrast), the LP relaxation would provide comparable results to those obtained using the mixed integer program solver, ZOOM. This is an important result because the run times associated with the LP relaxation were significantly less than those using ZOOM. In particular, if the rounding down policy is applied to the LP relaxation of the airlift model in this study, the resulting integer stockage level provides an objective function value which is within 1.87% of the LP upper bound or 1.73% of the upper bound established by the branch and bound procedure utilized by ZOOM. This is not quite as good as the result obtained

directly from ZOOM (within 0.07% of optimality), however, for those situations in which computer run time may be a consideration (e.g., as problem size increases) or in which very tight bounds may not be required, a very good solution can be obtained from the much simpler and less costly LP relaxation.

VI. CONCLUSIONS AND RECOMMENDATIONS

In this thesis, the problem of determining spare stockage levels for deployable MAGTFs was solved by

(a) stochastically modeling demand utilizing operational data from the Marine Corps Integrated Maintenance Management System, and then

(b) using mathematical programming to optimize a suitable, operationally relevant objective function.

To accomplish this, a new measure of effectiveness was defined which provides the operational commander on the ground with an objective indicator of the overall impact of the stockage policy on the probability of mission completion. This measure of effectiveness was evaluated as the end result of a probabilistic modeling step, a statistical data analysis, and a subsequent optimization.

The generality of the model formulation makes it adaptable to any of the three scenarios of present interest to the Marine Corps. The use of a PC-compatible algebraic modeling language (GAMS) makes it particularly easy to add or delete constraints as required. Specifically, the model and solution approach were applied to a realistic exercise scenario and an essentially optimal stockage policy was determined which maximized the probability of mission

completion within the constraints imposed by the deployment mode.

During the analysis of the MIMMS data base (i.e.,ERO History File) numerous problems and inconsistencies were encountered. As a result, the estimates of the failure rates for some of the secondary reparables are higher than would be the case if all the potential times to failure could be extracted from the data base. To overcome this discrepancy, it is highly recommended that the quarterly update procedure described in Chapter II be conducted at the Force Service Support Group level. If this cannot be done, then at the very least the meter readings associated with all pacing items should be recorded and input into MIMMS prior to each quarterly update. This would insure that beginning and ending meter readings are available for each serialized pacing item during the reporting period covered by the ERO History File and this would prevent the loss of valuable exposure time data.

The limited strategic lift assets available to deploying Marine Corps units demands that effective utilization be made of these resources. Since only a small percentage of these assets can be allocated for spare parts inventories, emphasis must be placed on stocking those parts which would have the greatest impact on the completion of the assigned mission. The reliability-based model developed in this

study accomplishes this objective by maximizing the probability of mission completion within the constraints of the mode of deployment.

APPENDIX A

LISTING OF SOURCE CODE FOR LANGRANGIAN RELAXATION

```
      PROGRAM RELAX2

      CHARACTER  NSN*13,NOMEN*7

      INTEGER  NCOMP,NEQUIP,NSPARE,MAXSPR,IDNUM,BSPARE,SPBND,NDENS,
     *NOPER,NXPOSE

      REAL*8   P,W,MAXWT,DINF,OPTDEV,ZLOW,ZUP,PSUC,HILAMD,WT,MTBF,
     *DURMIS,ZHLOW,XPOSE,ALPHA,BETA

      PARAMETER (NCOMP=25,NEQUIP=3,MAXSPR=10,MAXWT=44400.0D0,
     *DINF=1.0D20)

      DIMENSION  NSN(NCOMP),NOMEN(NCOMP),NSPARE(NCOMP),IDNUM(NCOMP),
     *P(NCOMP,0:MAXSPR),W(NCOMP,0:MAXSPR),BSPARE(NCOMP),SPBND(NCOMP),
     *NDENS(NCOMP),NOPER(NCOMP),XPOSE(NCOMP),NXPOSE(NCOMP),
     *DURMIS(NCOMP),ALPHA(NCOMP),BETA(NCOMP)

      CALL INIT2(DINF,MAXSPR,NEQUIP,MAXWT,P,W,NSN,NOMEN,HILAMD,NSPARE,
     *SPBND,NCOMP,IDNUM,NDENS,NOPER,XPOSE,NXPOSE,DURMIS,ALPHA,BETA)

      CALL BOUND(DINF,P,W,NCOMP,MAXSPR,SPBND,MAXWT,HILAMD,NSPARE,ZLOW,
     *ZUP,WT,BSPARE)

      CALL HERIST(DINF,P,W,NCOMP,MAXSPR,ZHLOW,WT,MAXWT,SPBND,BSPARE,
     *LARRAY)

      WRITE (10,50)
50    FORMAT ('1'/'0','----------------------------------------------',
     *'----------------')

      OPTDEV = -1.0D0 * 100.0D0 * (1.0D0 - (ZHLOW/ZUP))

      WRITE (10,100) ZHLOW
100   FORMAT (1X,'OBJECTIVE FUNCTION VALUE IS :',D17.10)
      WRITE (10,150) OPTDEV
150   FORMAT ('0','THIS SOLUTION IS WITHIN ',D17.10,' % OF THE OPTIMAL')
      PRINT *,'ZUP=',ZUP,'ZLOW=',ZLOW,'ZHLOW=',ZHLOW
      PSUC = EXP(ZHLOW)
      WRITE (10,175) PSUC
175   FORMAT ('0','P(MISSION SUCCESS) = ',F7.5)
      WRITE (10,180)
180   FORMAT ('0','-----------------------------------------------',
     *'------------')

      WRITE (10,200)
200   FORMAT ('0',5X,'NSN',10X,'END ITEM',6X,'SPARE LEVEL')

      DO 250 I=1,NCOMP
         WRITE (10,300) NSN(I),NOMEN(IDNUM(I)),BSPARE(I)
250   CONTINUE
300   FORMAT ('0',A13,7X,A7,7X,I4)

      PRINT *,'WEIGHT USED=',WT,'MAXWT=',MAXWT

      STOP
      END
```

```
      SUBROUTINE INIT2(DINF,MAXSPR,NEQUIP,MAXWT,P.W,NSN,NOMEN,HILAMD,
     *NSPARE,SPBND,NCOMP,IDNUM,NDENS,NOPER,XPOSE,NXPOSE,DURMIS,ALPHA,
     *BETA)

      INTEGER  NCOMP
      REAL*8   MAXWT

      INTEGER  IDNUM(NCOMP),NDENS(NCOMP),NOPER(NCOMP),LMT,MAXSPR,
     *NSPARE(NCOMP),SPBND(NCOMP),NEQUIP,NXPOSE(NCOMP)

      REAL*8   W(NCOMP,0:MAXSPR),DURMIS(NCOMP),DINF,P1,Q,BETA(NCOMP),
     *PTEMP1,P(NCOMP,0:MAXSPR),HILAMD,HLTEMP,CUMSUM,XPOSE(NCOMP),PP,
     *ALPHA(NCOMP)

      CHARACTER  NSN*13(NCOMP),NOMEN*7(NCOMP)

      READ (1,10) (NSN(I),IDNUM(I),XPOSE(I),NXPOSE(I),W(I,1),SPBND(I),
     *ALPHA(I),BETA(I), I=1,NCOMP)
10    FORMAT (A13,2X,I2,2X,F9.1,2X,I4,2X,F6.1,2X,I3,2X,F7.4,2X,F8.2)

      READ (2,20) (NOMEN(I),NDENS(I),NOPER(I),DURMIS(I), I=1,NEQUIP)
20    FORMAT (A7,4X,I3,4X,I3,4X,F6.1)

      DO 21 I=1,NCOMP
       PRINT *,NSN(I),IDNUM(I),XPOSE(I),NXPOSE(I),W(I,1),SPBND(I),
     *ALPHA(I),BETA(I)
21    CONTINUE

      DO 22 I=1,NEQUIP
       PRINT *,NOMEN(I),NDENS(I),NOPER(I),DURMIS(I)
22    CONTINUE


      DO 25 I=1,NCOMP
         NSPARE(I) = 0
         DO 50 J=0,MAXSPR
            P(I,J) = DINF
            IF (J .NE. 1) W(I,J) = DINF
50       CONTINUE
25    CONTINUE

      DO 75 I=1,NCOMP
         P1 = (XPOSE(I)+BETA(I))/(NDENS(IDNUM(I))*
     *DURMIS(IDNUM(I)) + XPOSE(I) + BETA(I))
         Q = 1.0D0 - P1
         PP = P1 ** (NXPOSE(I) + ALPHA(I))
         LMT = NDENS(IDNUM(I)) - NOPER(IDNUM(I))
         DO 100 J=0,SPBND(I)
            IF (J .NE. 1) W(I,J) = J*W(I,1)
            CUMSUM = 0.0D0
            DO 125 K=0,(LMT + J)
               IF (K .EQ. 0) THEN
                  PTEMP1 = 1.0D0
               ELSE
            PTEMP1 = PTEMP1*((((NXPOSE(I)+ALPHA(I))+(K-1))*1.0D0)/K)*Q
               END IF
               CUMSUM = CUMSUM + PTEMP1
125         CONTINUE
               CUMSUM = CUMSUM * PP
               P(I,J) = LOG(CUMSUM)
               PRINT *,'I=',I,'J=',J,'LN=',P(I,J),'CUMSUM=',CUMSUM
100      CONTINUE
75    CONTINUE
```

```
      HILAMD = -DINF
      DO 200 I=1,NCOMP
         DO 225 J=1,SPBND(I)
            HLTEMP = ( P(I,J) - P(I,0) ) / W(I,J)
            IF (HLTEMP .GT. HILAMD) HILAMD = HLTEMP
225      CONTINUE
200   CONTINUE
      PRINT *,'HILAMBDA=',HILAMD
      RETURN
      END


      SUBROUTINE BOUND(DINF,P,W,NCOMP,MAXSPR,SPBND,MAXWT,HILAMD,NSPARE,
     *ZLOW,ZUP,WT,BSPARE)

      INTEGER  NCOMP,BSPARE(NCOMP),MAXSPR,SPBND(NCOMP),NSPARE(NCOMP)

      REAL*8   BSTLAM,ZLOW.WT,P(NCOMP,0:MAXSPR),W(NCOMP,0:MAXSPR),DINF,
     *MAXWT,HILAMD,EPS,LFTEND,RTEND,ZLBEST,LAMBDA,ZUP


      EPS = HILAMD/1.0D04
      LFTEND = 0.0D0
      RTEND  = 1.1D0*HILAMD
      ZLBEST = -DINF
50    LAMBDA = (LFTEND + RTEND)/2.0D0

      CALL MAXFCT(DINF,P,W,NCOMP,MAXSPR,SPBND,MAXWT,LAMBDA,NSPARE,ZLOW,
     *ZUP,WT)

      IF (WT .LE. MAXWT) THEN
         RTEND = LAMBDA
         IF (ZLOW .GE. ZLBEST) THEN
            ZLBEST = ZLOW
            BSTLAM = LAMBDA
         END IF
      ELSE
         LFTEND = LAMBDA
      END IF

      IF ( (RTEND - LFTEND) .GT. EPS ) GOTO 50

      CALL MAXFEA(DINF,P,W,NCOMP,MAXSPR,SPBND,BSTLAM,BSPARE,WT,ZLOW)

      RETURN
      END
```

```
      SUBROUTINE MAXFCT(DINF,P,W,NCOMP,MAXSPR,SPBND,MAXWT,LAMBDA,NSPARE,
     *ZLOW,ZUP,WT)

      INTEGER  NCOMP,NSPARE(NCOMP),MAXSPR,SPBND(NCOMP),INDEX

      REAL*8   LAMBDA,ZLOW,WT,P(NCOMP,0:MAXSPR),W(NCOMP,0:MAXSPR),DINF,
     *CTOTAL,CMPMAX,ZVALJ,OBJFCT,OBJVAL,ZUP,MAXWT


      WT = 0.0D0
      CTOTAL = 0.0D0
      ZLOW = 0.0D0

      DO 25 I=1,NCOMP
         CMPMAX = -DINF
         INDEX = 0

         DO 50 J=0,SPBND(I)
            ZVALJ  = P(I,J) - LAMBDA*W(I,J)
            OBJVAL = P(I,J)
            IF (ZVALJ .GE. CMPMAX) THEN
               NSPARE(I) = J
               CMPMAX = ZVALJ
               OBJFCT = OBJVAL
               INDEX  = J
            END IF
50       CONTINUE
         CTOTAL = CTOTAL + CMPMAX
         ZLOW = ZLOW + OBJFCT
         WT = WT + W(I,INDEX)
25    CONTINUE

      ZUP = CTOTAL + LAMBDA*MAXWT

      RETURN
      END
      SUBROUTINE MAXFEA(DINF,P,W,NCOMP,MAXSPR,SPBND,BSTLAM,BSPARE,WT,
     *ZLOW)

      INTEGER  NCOMP,BSPARE(NCOMP),MAXSPR,SPBND(NCOMP),INDEX

      REAL*8   BSTLAM,ZLOW,WT,P(NCOMP,0:MAXSPR),W(NCOMP,0:MAXSPR),DINF,
     *CMPMAX,ZVALJ,OBJFCT,OBJVAL


      WT = 0.0D0
      ZLOW = 0.0D0

      DO 25 I=1,NCOMP
         CMPMAX = -DINF
         INDEX = 0

         DO 50 J=0,SPBND(I)
            ZVALJ  = P(I,J) - BSTLAM*W(I,J)
            OBJVAL = P(I,J)
            IF (ZVALJ .GE. CMPMAX) THEN
               BSPARE(I) = J
               CMPMAX = ZVALJ
               OBJFCT = OBJVAL
               INDEX  = J
            END IF
50       CONTINUE
         ZLOW = ZLOW + OBJFCT
         WT = WT + W(I,INDEX)
25    CONTINUE

      RETURN
      END
```

```
      SUBROUTINE HERIST(DINF,P,W,NCOMP,MAXSPR,ZHLOW,WT,MAXWT,SPBND,
     *BSPARE,LARRAY)

      INTEGER  NCOMP,BSPARE(NCOMP),MAXSPR,SPBND(NCOMP),INDEX

      REAL*8   DINF,P(NCOMP,0:MAXSPR),W(NCOMP,0:MAXSPR),WT1,WT,MAXWT,
     *XSSWT,CHKVAL,LARRAY(NCOMP),NUMER,DENOM,LBEST,ZHLOW

      CHKVAL = DINF/1.1D0
      WT1 = WT
25    XSSWT = MAXWT - WT1

      DO 50 I=1,NCOMP
         IF ((BSPARE(I) .GE. SPBND(I)) .OR. (P(I,(BSPARE(I)+1)) .GT.
     *CHKVAL)) THEN
            LARRAY(I) = -DINF
            GOTO 50
         END IF
         NUMER = P(I,(BSPARE(I)+1)) - P(I,BSPARE(I))
         DENOM = W(I,(BSPARE(I)+1)) - W(I,BSPARE(I))
         IF ((DENOM .GT. XSSWT) .OR. (NUMER .LE. 1.0D-10)) THEN
            LARRAY(I) = -DINF
            GOTO 50
         ELSE
            LARRAY(I) = NUMER/DENOM
         END IF
         IF (LARRAY(I) .LT. 0.0D0) LARRAY(I) = -LARRAY(I)
50    CONTINUE
      LBEST = -DINF
      DO 75 I=1,NCOMP
         IF (LARRAY(I) .GT. LBEST) THEN
            LBEST = LARRAY(I)
            INDEX = I
         END IF
75    CONTINUE
      IF (LBEST .LT. (-CHKVAL)) GOTO 100
      WT1 = WT1 - W(INDEX,BSPARE(INDEX)) + W(INDEX,(BSPARE(INDEX)+1))
      BSPARE(INDEX) = BSPARE(INDEX) + 1
      IF (WT1 .LE. MAXWT) GOTO 25
100   ZHLOW = 0.0D0
      WT = 0.0D0
      DO 125 I=1,NCOMP
         ZHLOW = ZHLOW + P(I,BSPARE(I))
         WT = WT +  W(I,BSPARE(I))
125   CONTINUE

      RETURN
      END
```

APPENDIX B

SAMPLE INPUT FILES

PRINCIPAL END ITEM FILE

```
M60A1       5     4     60.0
LVTP7A1    12    11     40.0
M101A1      5     4   2000.0
```

TABLE B-1

PRINCIPAL END ITEM FILE SPECIFICATIONS

LINE	COLUMN	DATA DESCRIPTION	DATA TYPE
All	1-7	Nomenclature of end item	Character
	8-11	Blank	
	12-14	Quantity of end item	Integer
	15-18	Blank	
	19-21	Number of end items required operational	Integer
	22-25	Blank	
	26-31	Mission duration	Real*8

SECONDARY REPARABLE COMPONENT FILE

2815001245387	1	2091.0	7	2000.0	5	0.34000	58.6903
2520002241867	1	2091.0	8	1000.0	5	1.00000	1.0000
2815002395819	1	2091.0	3	500.0	5	1.00000	1.0000
2520001549632	1	2091.0	5	500.0	5	0.10110	13.1850
2530000886657	1	2091.0	1	100.0	10	1.00000	1.0000
2520001184942	1	2091.0	1	250.0	5	1.00000	1.0000
2920010135802	1	2091.0	1	500.0	10	1.00000	1.0000
1010010703803	1	2091.0	1	1000.0	10	1.00000	1.0000
1025019388105	1	2091.0	5	100.0	5	1.00000	1.0000
1010010720397	1	2091.0	3	2000.0	5	1.00000	1.0000
1025010708993	1	2091.0	1	1000.0	5	1.00000	1.0000
2815004303480	2	3258.0	8	500.0	5	0.44330	142.9404
2520003973384	2	3258.0	23	1000.0	5	0.69700	69.0598
2520001443385	2	3258.0	14	2000.0	5	1.00000	1.0000
5805014593410	2	3258.0	10	250.0	5	0.20310	27.7348
2520008949532	2	3258.0	3	500.0	5	1.00000	1.0000
2910011714636	2	3258.0	1	100.0	10	1.00000	1.0000
2530011509757	2	3258.0	4	1000.0	10	1.00000	1.0000
5865012207848	2	3258.0	2	100.0	10	1.00000	1.0000
4320012035660	2	3258.0	3	500.0	10	1.00000	1.0000
2920002317276	2	3258.0	1	500.0	10	1.00000	1.0000
2530000886650	2	3258.0	1	250.0	5	1.00000	1.0000
1005011855059	3	20996.0	3	1000.0	5	1.00000	1.0000
1005011086434	3	20996.0	11	250.0	5	1.00000	1.0000
1005011457709	3	20996.0	7	500.0	5	1.00000	1.0000

TABLE B-2

SECONDARY REPARABLE/COMPONENT FILE SPECIFICATIONS

LINE	COLUMN	DATA DESCRIPTION	DATA TYPE
All	1-13	National Stock Number (NSN)	Character
	14-15	Blank	
	16-17	Identification Number (1 for M60A1, 2 for LVTP7A1, 3 for M101A1)	Integer
	18-19	Blank	
	20-28	Total exposure time for component	Real*8
	29-30	Blank	
	31-34	Number of observed failures for component	Integer
	35-36	Blank	
	37-42	Component shipping weight	Real*8
	43-44	Blank	
	45-47	Maximum possible stockage for component	Integer
	48-49	Blank	
	50-56	Shape parameter of prior distribution for component failure rate	Real*8
	57-58	Blank	
	59-66	Scale parameter of prior distribution for component failure rate	Real*8

APPENDIX C

SAMPLE OUTPUT FILE

OBJECTIVE FUNCTION VALUE IS :-0.8820263158D+00

THIS SOLUTION IS WITHIN 0.4761377367D+00 % OF THE OPTIMAL

P(MISSION SUCCESS) = 0.41394

NSN	END ITEM	SPARE LEVEL
2815001245387	M60A1	2
2520002241867	M60A1	3
2815002395819	M60A1	2
2520001549632	M60A1	2
2530000886657	M60A1	3
2520001184942	M60A1	2
2920010135802	M60A1	1
1010010703803	M60A1	1
1025019338105	M60A1	5
1010010720397	M60A1	1
1025010708993	M60A1	1
2815004303480	LVTP7A1	4
2520003973384	LVTP7A1	5
2520001443385	LVTP7A1	4
5805014593410	LVTP7A1	5
2520008949532	LVTP7A1	2
2910011714636	LVTP7A1	3
2530011509757	LVTP7A1	2
5865012207848	LVTP7A1	3
4320012035660	LVTP7A1	2
2920002317276	LVTP7A1	1
2530000886650	LVTP7A1	2
1005011855059	M101A1	4
1005011086434	M101A1	5
1005011457709	M101A1	5

APPENDIX D

GAMS SOURCE CODE

```
SETS
    I   end item               /E1875,E0346,E0640/
    J   component              /1*25/
    K   spare stockage level   /S0*S10/
    L   dummy index            /D0*D11/;

SCALARS
    WTCAP  total weight capacity (lbs) /44400/
    PSUC   probability of mission completion;

PARAMETERS
    EQDENS(I)  density of end item i
      /E1875      5
       E0846     12
       E0640      5/;

PARAMETERS
    NOPER(I)  number end item i required operational during mission
      /E1875      4
       E0846     11
       E0640      4/;

PARAMETERS
    DURMIS(I)  mission duration for end item i (hours rounds miles)
      /E1875     60
       E0846     40
       E0640   2000/;

PARAMETERS
    SHIPWT(J)  shipping weight of component j (lbs)
      /1       2000
       2       1000
       3        500
       4        500
       5        100
       6        250
       7        500
       8       1000
       9        100
       10      2000
       11      1000
       12       500
       13      1000
       14      2000
       15       250
       16       500
       17       100
       18      1000
       19       100
       20       500
       21       500
       22       250
       23      1000
       24       250
       25       500/;
```

```
PARAMETERS
    SPRMAX(J)  max number of spares allowed for component j
       /1        5
        2        5
        3        5
        4        5
        5       10
        6        5
        7       10
        8       10
        9        5
       10        5
       11        5
       12        5
       13        5
       14        5
       15        5
       16        5
       17       10
       18       10
       19       10
       20       10
       21       10
       22        5
       23        5
       24        5
       25        5/;

PARAMETERS
    NOBS(J)  number of data points observed for component j
       /1        7
        2        8
        3        3
        4        5
        5        1
        6        1
        7        1
        8        1
        9        5
       10        3
       11        1
       12        8
       13       23
       14       14
       15       10
       16        3
       17        1
       18        4
       19        2
       20        3
       21        1
       22        1
       23        3
       24       11
       25        7/;
```

```
PARAMETERS
    ALPHA(J)  shape paramtr for Gamma prior distribution
        /1    0.34
         2     1
         3     1
         4    0.1011
         5     1
         6     1
         7     1
         8     1
         9     1
        10     1
        11     1
        12    0.4433
        13    0.6970
        14     1
        15    0.2031
        16     1
        17     1
        18     1
        19     1
        20     1
        21     1
        22     1
        23     1
        24     1
        25     1/;

PARAMETERS
    BETA(J)   scale paramtr for Gamma prior distribution
        /1      58.6903
         2       1
         3       1
         4      13.1850
         5       1
         6       1
         7       1
         8       1
         9       1
        10       1
        11       1
        12     142.9404
        13      69.0598
        14       1
        15      27.7348
        16       1
        17       1
        18       1
        19       1
        20       1
        21       1
        22       1
        23       1
        24       1
        25       1/;
```

```
PARAMETERS
    OK(I,J)  equals 1 if component j is a component of end item i
       /E1875.(1*11)      1
        E0846.(12*22)     1
        E0640.(23*25)     1/;

PARAMETERS
    TXPOSE(I)  total observed exposure for end item i (hrs rds mi)
       /E1875      2091
        E0846      3258
        E0640     20996/;

PARAMETER
    LMT(I,J)  max number of failures for component j;
    LMT(I,J)$( OK(I,J) EQ 1 ) = EQDENS(I) - NOPER(I);

SETS
    OKLVL1(I,J,K)  feasible spare stockage levels for component j;
    OKLVL1(I,J,K)$((ORD(K)-1 LE SPRMAX(J)) AND (OK(I,J) EQ 1)) = YES;

*  The following set is necessary to obtain the cumulative distribution
*  function of the negative binomial density.

SETS
  OKLVL2(I,J,K,L);
  OKLVL2(I,J,K,L)$( (ORD(K)-1 LE SPRMAX(J)) AND (OK(I,J)EQ 1) AND
                 (ORD(L)-1 LE LMT(I,J)+ORD(K)-1) ) = YES;

DISPLAY OKLVL2;

OPTIONS DECIMALS = 8;

PARAMETER
    P(I,J)        negative binomial paramtr
    Q(I,J)        one minus the p paramtr
    PP(I,J)       p paramtr raised to power of nobs plus alpha
    TWT(I,J,K)    total snipping wt of k spares of component j
    C(I,J,K)      cumulative density for negative binomial
    F(I,J,K)      natural logarithm of cumulative density
    PTMP(I,J,K,L)  temporary paramtr needed to sum cdf recursively;

 P(I,J)$(OK(I,J) EQ 1) =
 (TXPOSE(I) + BETA(J))/((EQDENS(I)*DURMIS(I)) + TXPOSE(I) + BETA(J));

 Q(I,J)$(OK(I,J) EQ 1) = 1 - P(I,J);

 PP(I,J)$(OK(I,J) EQ 1) = P(I,J) ** (NOBS(J) + ALPHA(J));

 TWT(I,J,K)$OKLVL1(I,J,K) = SHIPWT(J) * (ORD(K) - 1);

 PTMP(I,J,K,"D0")$OKLVL1(I,J,K) = 1;

*  The following LOOP function is used to calculate the cdf from the
*  negative binomial pdf recursively.
```

```
 LOOP(L, PTMP(I,J,K,L+1)$OKLVL2(I,J,K,L) =
 Q(I,J)*((NOBS(J)+ALPHA(J)+ORD(L)-1)/(ORD(L)))*PTMP(I,J,K,L) );

 C(I,J,K)$OKLVL1(I,J,K) =
SUM(L$((ORD(L)-1) LE (LMT(I,J)+ORD(K)-1)), PTMP(I,J,K,L))*PP(I,J);

 F(I,J,K)$OKLVL1(I,J,K) = LOG( C(I,J,K) );

 DISPLAY F;

VARIABLES
    X(I,J,K)  1 if stock k spares for component j of end item i
    MOE       natural log of probability of mission completion;

BINARY VARIABLE X;

EQUATIONS
    WTCONST       weight constraint equation
    SELECT(I,J)   select exactly 1 of the possible stockage levels
    MOEDEF        measure of effectiveness definition;

WTCONST..  SUM((I,J,K)$OKLVL1(I,J,K), TWT(I,J,K)*X(I,J,K)) =L= WTCAP;

 SELECT(I,J)$( OK(I,J) EQ 1 )..
            SUM( K$OKLVL1(I,J,K), X(I,J,K) ) =E= 1;

 MOEDEF..  SUM((I,J,K)$OKLVL1(I,J,K), F(I,J,K)*X(I,J,K) ) =E= MOE;

 MODEL STOCK /ALL/;

 OPTIONS OPTCR = 0.0015, OPTCA = 0;

 SOLVE STOCK USING MIP MAXIMIZING MOE;

 PSUC = EXP( MOE.L )

DISPLAY X.L,MOE.L,PSUC;
```

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